Spectrum Crack Growth Analysis Using the Willenborg Model

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This paper presents an evaluation of the Willenborg crack growth retardation model. This model utilizes an "effective stress" concept to reduce the crack tip stress intensity factor. With the exception of the constant amplitude growth rate data for a given material, the model does not rely on any empirically derived parameters. A summary of the model is presented, including the basic assumptions, the range of application, and usage limitations. Comparisons with existing experimental data for typical randomized block spectra and flight-by-flight spectra show very good correlation. However, the model does not produce good results for highly ordered (low-high or high-low) spectra. Results are also presented for single and multiple overload spectra for 7075-TG and 2024-T3 aluminum alloys. The periodic overload behavior for these alloys was predicted with an accuracy that was within the typical scatter of the constant amplitude growth rate data.

Nomenclature

a	= crack length, in.
a_l	= crack length at which overload is applied, in.
a_2	= crack length at which retardation ceases for σ_2, in.
a_c	= any crack length within overload affected zone, in.
a_p	= overload affected crack length, in.
da/dN	= crack growth rate, in./cycle
K	= stress intensity factor, ksi-in.$^{1/2}$
ΔK	= stress intensity factor range, $K_{max} - K_{min}$, ksi – in.$^{1/2}$
N	= cycles
R	= stress ratio, $\sigma_{min}/\sigma_{max}$
R_{yl}	= size of yield zone due to σ_1 applied at a_l, in.
R_{y2}	= size of yield zone due to σ_2 applied at a_c, in.
σ_l	= overload stress, ksi
σ_2	= maximum stress following overload, ksi
σ_3	= minimum stress following overload, ksi
σ_{ap}	= stress required to achieve a_p, ksi
σ_{res}	= residual stress, ksi

Introduction

CURRENT predictive analysis techniques for crack propagation under cyclic loading rely on the direct integration of basic constant amplitude growth rate data derived from laboratory tests in simple coupons. Many computer routines exist which perform this direct integration on each discrete load level and accumulate the crack growth. Such an automated procedure is the CRACKS II computer program developed at the Flight Dynamics Laboratory.[1]

Variations between predicted and actual test lives have been noted for cases of variable amplitude spectrum loading.[2,3] These differences have been attributed to the interaction between high stress amplitudes (overloads) and the following lower stress cycles, as shown in Fig. 1. The effect of these overloads is to retard the succeeding growth rates over a given crack length. Neglecting these interaction effects can result in grossly conservative predictions of crack growth life.

Several attempts at developing mathematical models for crack growth retardation have been made. Wheeler[4] developed a retardation factor C_p, which operates directly by reducing the crack growth rate da/dN. This procedure requires previous spectrum growth data to derive a retardation exponent m by curve fitting spectrum test data. Moderate success has been achieved by this technique. Elber[5] advanced the concept of crack closure. He postulates that crack propagation takes place only when the crack tip is fully open. This model develops an effective stress intensity factor range, ΔK_{eff} based on the difference between the maximum applied stress and the stress required to open the crack. The primary drawback to this model is the lack of information concerning the behavior of the closure stress.

Willenborg Model

The driving force behind the development of the Willenborg model was the desire to obtain a load interaction (retardation) concept that relies solely on the constant amplitude crack growth rate data. Willenborg[6] assumed that retardation is proportional to a reduction in the maximum applied stress due to residual stresses set up by a preceding overload. An effective ΔK is determined by assuming a form for the residual stress σ_{res}, present at the crack tip and reducing the applied stresses by this amount. Once obtained, this effective ΔK is used in conjunction with constant amplitude growth rate data to determine the increment of crack growth. Following the yield zone interaction concept of Wheeler, as shown in Fig. 2, the residual stress is assumed to decay over a crack length equal to the plastic zone R_y created by the overload. The expression for R_y is, from Irwin

$$R_y = (1/\alpha\pi)\ (K/\sigma_y)^2 \quad (1)$$

where $\alpha = 2$ for plane stress and $\alpha = 6$ for plane strain, and σ_y is the material yield stress. Figure 3 summarizes the operation of the Willenborg model. The residual stress is seen to be the difference between the maximum applied stress σ_2, and σ_{ap}, the stress required to achieve no retardation. A full development of the model is given in Ref. 6.

The Willenborg model predicts that an overload σ_1, which is twice the magnitude of the following maximum stress σ_2, will cause crack arrest (i.e., the effective ΔK will be zero). The ratio σ_1/σ_2 is called the overload ratio. Recent experimental work[7] indicates that, for aluminum, an overload ratio of approximately 2.3 will cause crack arrest, whereas for titanium, an overload ratio of 2.8 is required. A modification[8] to the model which accounts for this overload ratio effect has been incorporated which produces improved results for spectrum loading.

CRACKS II Program

The CRACKS II computer program[1] is a specialized integration routine, which determines incremental crack growth for each of a series of discrete load levels, given a stress intensity factor formulation and crack growth rate relationship. CRACKS II provides the additional capability of modeling

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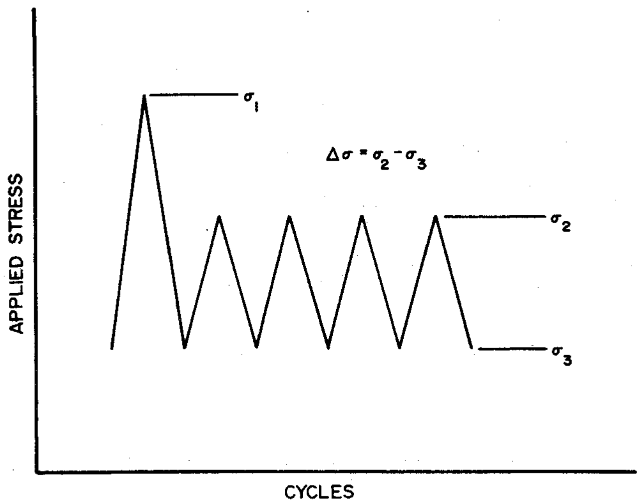

Fig. 1 Loading conditions for retardation.

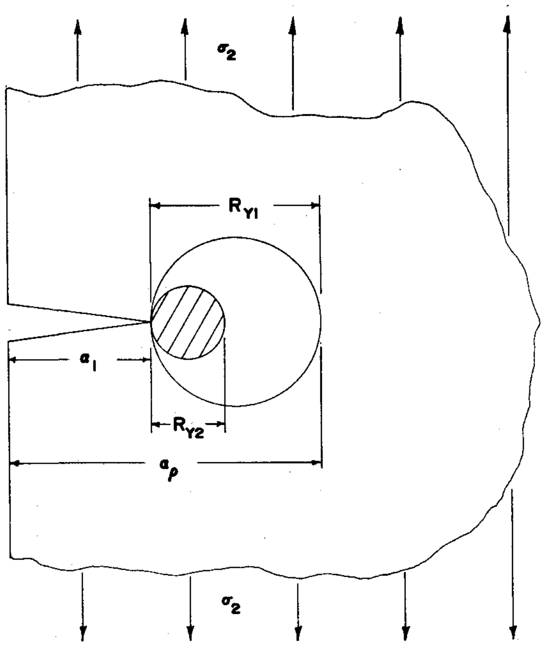

Fig. 2 Yield zone interaction concept.

load interaction effects (crack growth retardation). At the present time, three models are available; the Wheeler model,[4] the Willenborg model,[6] and a newly developed closure model.[9] Further flexibility is provided through three crack growth rate relationships

Paris $$\mathrm{d}a/\mathrm{d}N = C(\Delta K)^{n} \tag{2}$$

Forman $$\mathrm{d}a/\mathrm{d}N = \frac{C(\Delta K)^{n}}{(1-R)K_c - \Delta K} \tag{3}$$

Walker $$\mathrm{d}a/\mathrm{d}N = C[(1-R)^{m}K_{max}]^{n} \tag{4}$$

where C, K_c, m, and n are empirical constants. At present the program also provides nine stress intensity factor formulations ranging from a center cracked finite width sheet to a corner flaw from a fastener hole.

For each load level in the spectrum, CRACKS II determines the stress intensity K_{max}, or the stress intensity range ΔK, corresponding to the current crack length. These values then are modified by the load interaction model to account for retardation effects. The resulting "effective" K_{max} or ΔK then is used in conjunction with one of the preceding growth rate relationships to determine the incremental crack growth. The

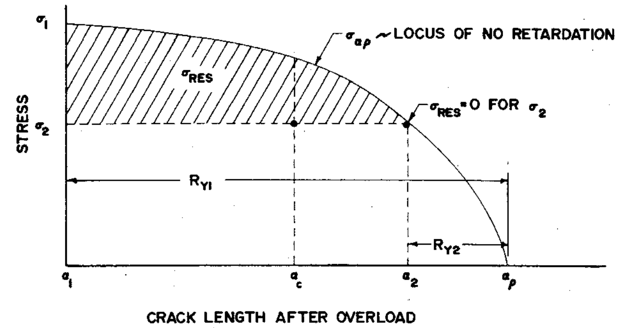

Fig. 3 Residual stress decay in the Willenborg model.

crack growth increments then are summed to give crack growth vs cycles.

Correlation with Test Data

The validity of any predictive model can be established only through correlation with test data for a wide variety of conditions. This evaluation of the Willenborg model was conducted in stages; beginning with simple two-level block loading on center-cracked panels and extending to flight-by-flight loading on flawed fastener holes. The test data all were taken from reports available in the open literature. Where constant amplitude crack growth rate data for the given material were furnished, they were used directly. When the growth rate data were not published, average values from handbooks were used.

Two-Level Block Loading

The simplest spectrum that will introduce retardation effects is a two-level repeating block spectrum, such as that shown in Fig. 1. The first step in increasing the complexity of the spectrum is to increase the number of overloads σ_1. Defining M as the number of overload cycles and N as the number of cycles of the lower stress permits the characterization of any two-level spectrum in terms of M and N. For example, a repeating block consisting of ten cycles of σ_1 followed by fifty cycles of σ_2 could be represented by $M=10$ and $N=50$. Test data of this type were presented in Ref. 3 for center-cracked panels. Correlation for single overload ($M=1$) blocks applied to 2024-T3 aluminum panels is shown in Fig. 4. Similar correlation for multiple overload blocks applied to 7075-T6 aluminum panels is shown in Fig. 5. Both of these sets of analytical predictions were conducted using average crack growth rate data available in the open literature. The Forman equation [Eq. (3)] was used to represent the crack growth rate relationship.

Blocked Spectrum Loading

The next level of complexity attempted in the evaluation of the Willenborg model was a blocked spectrum representing actual aircraft usage. The spectrum selected was a 58 level randomized block representing 200 hr of aircraft usage for the F-111 aircraft. A very extensive test program was reported in Ref. 10, which used surface flawed specimens made of D6ac steel. Several variations of the basic 200-hr block were examined which included both stress level variations and ordering of stresses. A key to these variations is presented in Table 1.

Table 1 F-111 spectrum variations

Key	Variation
A	Basic spectrum (σ_{max} = 106ksi)
A1	Low to high order
A2	High to low order
A3	Truncated spectrum (σ_{max} = 100ksi)
A4	Truncated spectrum (σ_{max} = 91ksi)
A5	Basic spectrum (all stresses increased 10%)

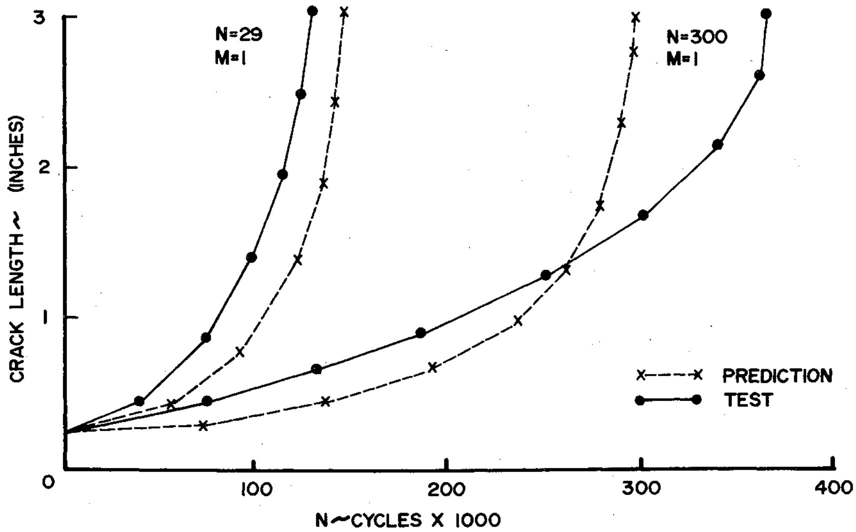

Fig. 4 Single overload correlation, 2024-T3–Ref. 3.

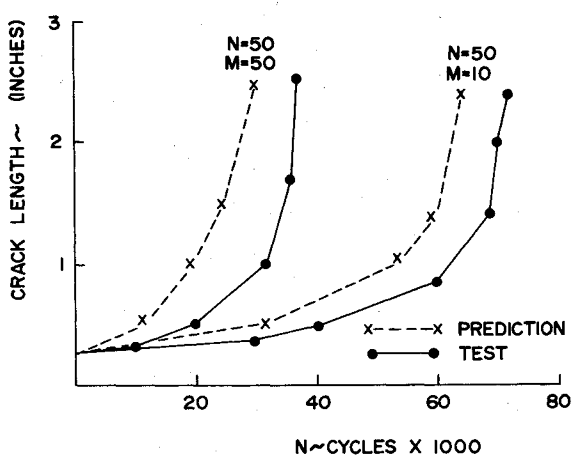

Fig. 5 Multiple overload correlation, 7075-T6 – Ref. 3.

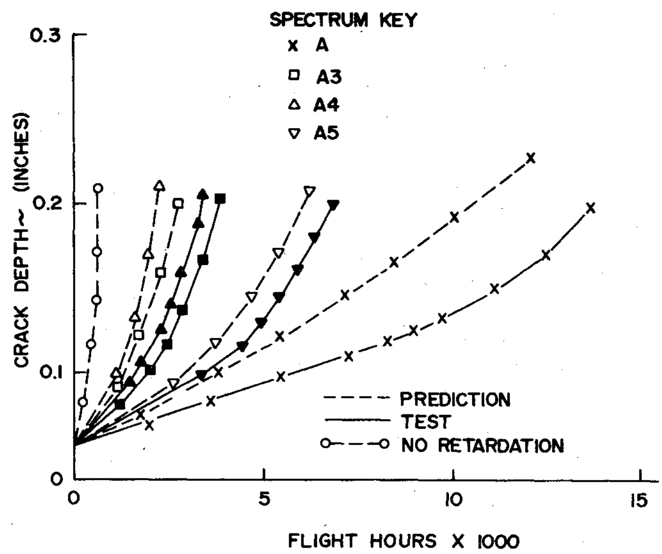

Fig. 6 F-111 Spectrum, stress level variations – Ref. 10.

Again using Forman's equation to characterize the crack growth rate relationship with the constants taken from Ref. 10, the Willenborg model was used to predict the crack growth behavior for each of the variations in the basic spectrum. As shown in Fig. 6, the model predicts the trends well for all the stress level variations tested in Ref. 10. It is particularly important to note the extreme degree of conservatism which would be introduced by neglecting retardation effects in the analysis. Figure 7 depicts predictions for the ordered spectra tested in Ref. 10. The operation of the Willenborg model is such that the influence of the high loads is overemphasized when all of the stresses are ordered in each block. In the low to high sequence (A1), the maximum load appears at the end of each block, causing it to dominate the lower stresses in the next block completely. Hence, the model predicts an excessive amount of retardation. Conversely, in the high to low sequence, the effect of the maximum load is diminished because all of the high stresses in the block are bunched immediately after the highest load. In the Willenborg model, this sequence produces very little retardation.

Flight-by-Flight Spectrum Loading

The most realistic loading environment is the flight-by-flight spectrum wherein the loads corresponding to each flight of the aircraft are applied separately, rather than lumped into some arbitrary number of flight hours. Two approaches for defining flight-by-flight spectra are commonly used. The first is to define an "average" flight, which becomes the baseline flight, and insert higher loads as they occur (i.e., once per ten flights, once per hundred, etc.). This approach will be called the blocked flight-by-flight spectrum. The second approach defines typical missions (e.g., low-altitude resupply) and constructs a flight-by-flight profile based upon a defined mission sequence. Reference 11 presents experimental data for each of these types of flight-by-flight spectra for flawed fastener holes in 7075-T6511 aluminum. Figure 8 shows the Willenborg model correlation for a blocked flight-by-flight spectrum. The cross-hatching represents the difference in predictions due to the use of upper-bound and lower-bound crack growth rate data for the humid air environment. Again note the extreme conservatism of the prediction ignoring retardation effects. Figure 9 presents a comparison of the Willenborg model and test data from Ref. 11 for the C-5A fifteen-mission spectrum. Predictions are given for upper-bound and lower-bound growth rate data. Again the test data fall within the band defined by the scatter in crack growth rate data for the test environment.

A typical example of the type of flight-by-flight spectrum currently being used to evaluate crack growth characteristics of aircraft materials and structural components is described in Ref. 12. This spectrum is exceedingly complex; consisting of 9135 stress levels per 1000 flight hours. The component tested was a corner flaw from a fastener hole. The crack growth rate relationship took the form of the Walker equation [Eq. (4)]. Figure 10 compares the Willenborg model prediction and test data from Ref. 12. The correlation is remarkably good,

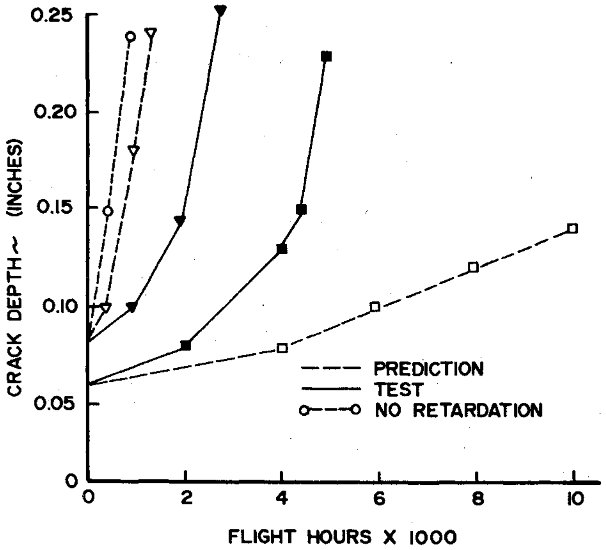

Fig. 7 F-111 spectrum, ordering variations – Ref. 10. Spectrum key: □-A1; ▽-A2.

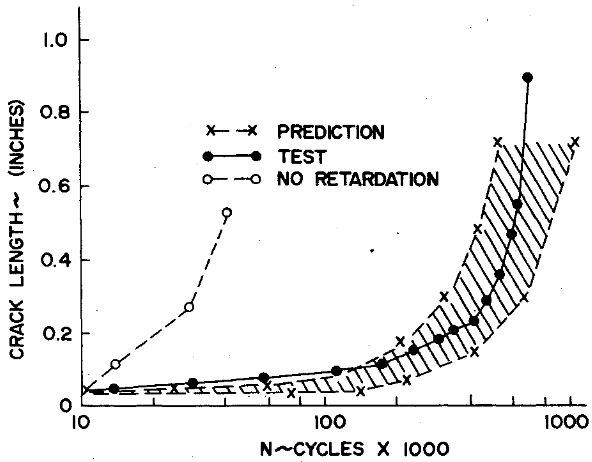

Fig. 8 C-5A blocked flight-by-flight spectrum – Ref. 11.

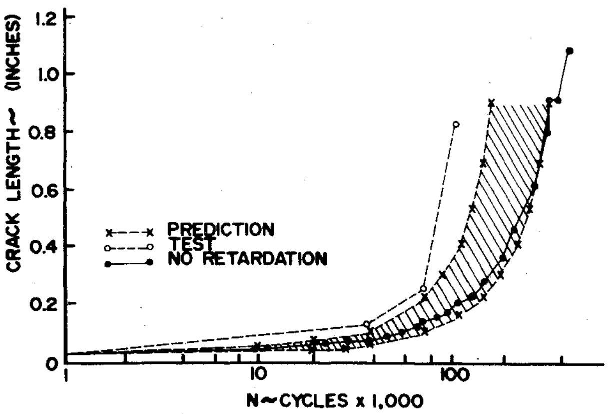

Fig. 9 C-5A flight-by-flight spectrum – Ref. 11.

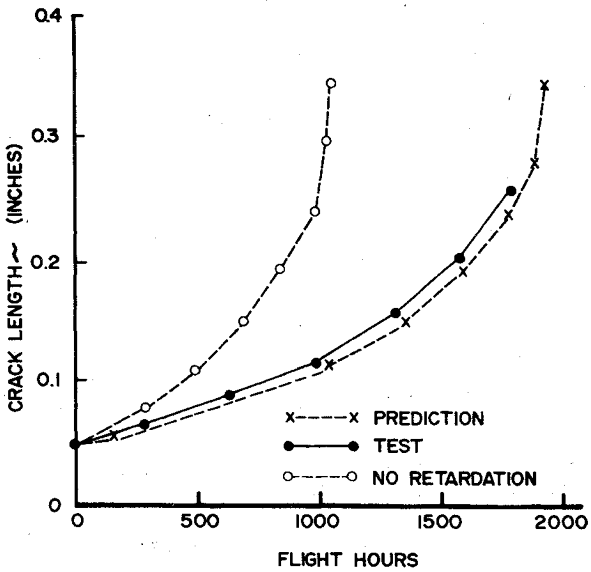

Fig. 10 F-4E flight-by-flight spectrum – Ref. 12.

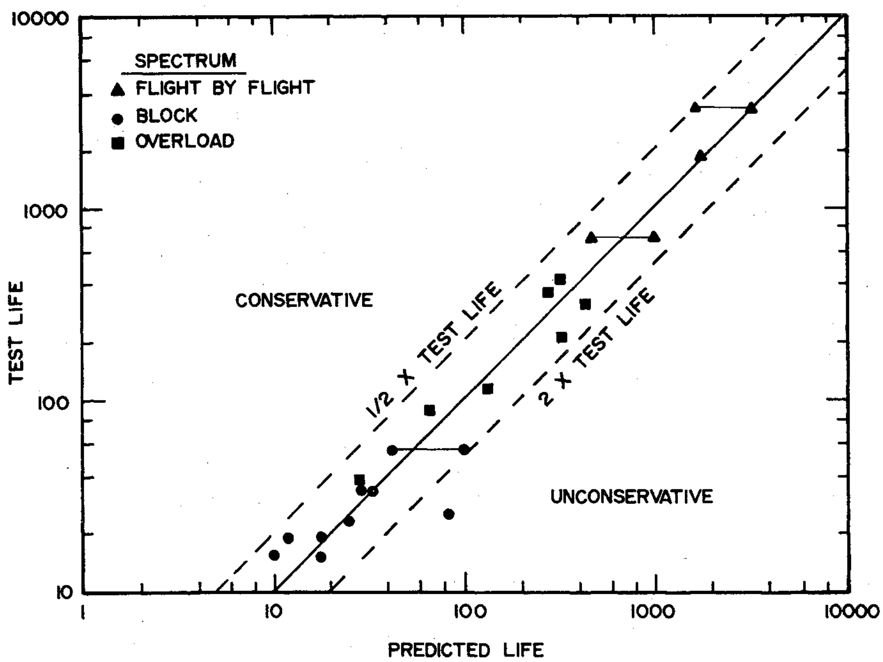

Fig. 11 Willenborg model correlation summary.

especially when compared with the solution neglecting retardation effects.

Limitations of the Willenborg Model

The correlation for the many types of spectra discussed in the preceding has, in general, been very good. However, the model, in its present form, has some significant shortcomings. The foremost of these is in the area of the compression effects. The Willenborg model ignores all portions of a stress level which are less than zero. This can be significant when a compressive load follows an overload. Some researchers[13] have shown that this sequence can diminish or completely negate the delay effects of the overload. Another shortcoming stems from the fact that the model treats each overload as a single discrete event. Hence, any cumulative effects of multiple overloads, such as those described in Ref. 14, are not accounted for. Another effect which the model does not exhibit is delayed retardation.[13,14] The maximum retardation predicted by the model occurs immediately after the application of the overload.

Conclusions

The Willenborg model is capable of making reasonable life predictions for crack propagation under typical aircraft-type spectra of a randomized nature. However, care should be taken when analyzing block spectrum loading, since this type of spectra maximizes the influence of high loads. Ordered block spectra should not be analyzed using this model. Figure 11 presents an overall look at the effectiveness of the model. This figure presents test data correlation for 21 tests representing five materials, four specimen goemetries, four environments, and 19 different spectra. The horizontal bars indicate the C-5A correlations that considered scatter in crack growth rate data. A most significant feature of these points is that the test data always fall within the scatter band generated by the constant amplitude crack growth rate data variations. Until something can be done to narrow this band, it will be difficult to pinpoint quantitative deficiencies in the correlation of any retardation model

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